

Pseudo-Dirac bino dark matter

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Abstract

While the bino-dominated lightest neutralino of the minimal supersymmetric Standard Model (MSSM) is an interesting and widely-studied candidate of the dark matter, the p -wave suppression of its annihilation cross section requires fine-tunings of the MSSM spectra to be consistent with Wilkinson Microwave Anisotropy Probe (WMAP) observations. We propose a pseudo-Dirac bino that arises in theories with D-type supersymmetry-breaking as an intriguing alternative candidate of dark matter. The pseudo-Dirac nature of the bino gives a natural mechanism of enhanced co-annihilation because these two states are degenerate in the absence of electroweak symmetry breaking. In addition, the lightest state can be consistent with limits of direct detection experiments because of the lack of vector interactions, as with the case of the MSSM bino.

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I. INTRODUCTION

The existence of dark matter is one of the direct evidences of physics beyond the standard model (SM), and supersymmetry (SUSY) is a strong candidate of such new physics. One the many virtues of the minimal supersymmetric Standard Model (MSSM) is that the lightest superpartner can serve as dark matter of the Universe. The lightest neutralino of the MSSM is an interesting and widely-explored candidate of dark matter ([1][2][3][4][5][6][7] and references therein). Because of electroweak symmetry breaking (EWSB), the lightest neutralino, χ_1^0 , is linear combination of the bino (λ_1), the wino (λ_2), and the Higgsinos ($\tilde{H}_{u,d}$). Expressing χ_1^0 as

$$\chi_1^0 = Z_{11}\lambda_1 + Z_{12}\lambda_2 + Z_{13}\tilde{H}_d + Z_{14}\tilde{H}_u, \quad (1)$$

where Z_{ij} are elements of the transformation matrix that diagonalizes the MSSM neutralino mass matrix, there are several important limits of interest. In the Higgsino-dominated ($Z_{13} \simeq Z_{14} \gg Z_{11}, Z_{12}$) and the wino-dominated ($Z_{12} \gg Z_{11}, Z_{13}, Z_{14}$) limits, χ_1^0 is typically nearly degenerate with either the charged Higgsinos and/or the charged winos, and the efficient self- and co-annihilation processes leads to a relic density that is less than the WMAP observation [8]

$$\Omega h^2 = 0.1195 \pm 0.0094. \quad (2)$$

On the other hand, in the bino-dominated limit ($Z_{11} \gg Z_{12}, Z_{13}, Z_{14}$), the annihilation cross section is p -wave suppressed, and bino relic density is typically higher than the WMAP observation.

For the relic density of χ_1^0 to be consistent with current observations, there typically requires fine-tunings of the MSSM spectra such that one or more of the following occurs [9]:

- Enhanced co-annihilation between χ_1^0 and another superpartner, typically the s-tau ($\tilde{\tau}$), when these two states are tuned to be nearly degenerate on the order of 1%.
- Enhanced s -channel resonance in the χ_1^0 annihilation cross section when the mass of one of the Higgs bosons is tuned to be close to twice the mass of χ_1^0 .
- Enhanced annihilation cross section from the wino/Higgsino mixture of χ_1^0 and enhanced co-annihilation of the charginos when either the mass of the winos (both neu-

tral and charged) or the Higgsinos (both neutral and charged) is tuned to be close to the bino mass [10].

The root of the problem is the p -wave suppression of the bino annihilation cross section due to the Majorana nature of the bino. On the other hand, Dirac particles carrying $SU(2)_L$ or $U(1)_Y$ quantum numbers such as the Kaluza-Klein (KK) neutrino of the minimal universal extra dimension model [11][12] are typically ruled out as dark matter by direct detection experiments such the Cryogenic Dark Matter Search (CDMS II)[13] and the XENON10 Dark Matter Experiment [14]. (For models where Dirac particles serve as viable dark matter, see Refs. [15][16].) We would like a natural mechanism that enhances the annihilation cross section, and at the same time, is consistent with the bounds of direct detection. The pseudo-Dirac bino is one such example. Pseudo-Dirac bino may arise in models of D-type SUSY-breaking. However, existing models [17] predict a heavy pseudo-Dirac bino with masses at least of the order of 1 TeV. In this paper, we consider pseudo-Dirac bino as a candidate of dark matter. Without effects of EWSB, the bino is a Dirac particle whose annihilation cross section is not p -wave suppressed and can naturally lead to observed relic density. When EWSB effects are considered, the Dirac bino splits into two nearly-degenerate Majorana states, and the annihilation of the lightest state is enhanced by co-annihilation between the these two nearly-degenerate bino states. On the other hand, the masses of these two states are separated by a few GeV's while the scale of momentum transfer in direct detection experiments is of the order of keV's. Therefore, the direct detection experiments is only sensitive to the lightest, Majorana, state whose cross section with nuclei is suppressed due to the lack of vector-current interactions. It is worth pointing out that this mechanism of suppressing the rates of direct detection operates as long as the splitting is larger than 10s of keV's, and is not limited to the splitting of a few GeV's (which happens to be our case here). For a similar idea involving the sneutrino as dark matter, see Reference [18].

In this paper we take a phenomenological approach, without appealing to a complete framework, and perform a simplified analysis of the relic density and direct detection rates of pseudo-Dirac bino dark matter. In Section II, we describe the relevant ingredients of D-type SUSY-breaking that lead to the pseudo-Dirac bino as dark matter. In Section III, we calculate the relic density and direct detection rates of pure-Dirac and pseudo-Dirac bino dark matter, and compare the results to those in MSSM. Finally, we summarize our results in Section IV.

II. D-TYPE GAUGE MEDIATED SUPERSYMMETRY BREAKING MODEL

We assume that SUSY-breaking originates in a hidden sector that contains a gauged $U(1)_X$ group that develops a non-zero $\langle D_X \rangle$, as well as non-zero $\langle F_Y \rangle$ for some field(s) Y that may or may not be charged under the $U(1)_X$ group (but neutral under SM gauge group). In general, both $\langle D_X \rangle$ and $\langle F_Y \rangle$ are communicated to the visible MSSM sector. Upon integrating out the messengers at the mass scale M , the Majorana gaugino masses are generated through the effective operator

$$\mathcal{L} \sim \int d^2\theta \frac{Y}{M} W_\alpha W^\alpha + \text{h.c.}, \quad (3)$$

where W_α is the chiral superfield containing the MSSM gaugino and gauge bosons, while MSSM scalar masses are generated through the effective operator

$$\mathcal{L} \sim \int d^4\theta \frac{Y^\dagger Y}{M^2} Q^\dagger Q. \quad (4)$$

In our phenomenological approach, we assume that Y is charged under $U(1)_X$, and thus the effective operator of Eq. (3) is not generated, while MSSM scalar soft masses are still generated through the operator in Eq. (4). This can be achieved, for example, by the charge assignments of the messengers and the hidden sector particle content under the $U(1)_X$ gauge group. Although this does not solve the flavor problem of the MSSM, we will take this as our starting point for the purpose of discussing pseudo-Dirac bino as dark matter. With the above assumptions of SUSY-breaking, the gauginos of the MSSM receive Dirac masses rather than Majorana masses. As a Dirac fermion contains more degrees of freedom than a Majorana fermion, additional fermionic states (the gaugino partners, denoted by ξ) that transform as adjoints of the SM group must be introduced. Supersymmetry (SUSY) then requires additional bosonic states (the s-gaugino, denoted by η) that also transform as adjoints of the SM group. The effective operator obtained by integrating out the messengers that gives a Dirac gaugino mass is

$$\mathcal{L} \sim \int d^2\theta \frac{X_\alpha}{M} \text{Tr} [W^\alpha \Xi] + \text{h.c.}, \quad (5)$$

where M is the mass scale of the messengers, and Ξ is a chiral superfield containing η and ξ . We can forbid Majorana masses for the gaugino partners of the form

$$\mathcal{L} \sim \int d^2\theta M \text{Tr} [\Xi \Xi] + \text{h.c.}, \quad (6)$$

by $U(1)_R$ symmetry that assigns the vector superfields W_α and X_α to have $U(1)_R$ charge of $+1$ and Ξ to have a zero $U(1)_R$ charge. Since the superpotential needs to have a $U(1)_R$ charge of $+2$, the effective operator of Eq. (5) is allowed by $U(1)_R$, while the operator of Eq. (6) is forbidden. We assume that the Dirac gaugino masses and the soft scalar masses (for both the MSSM superpartners and the s-gaugino) are all of the same scale of the order 1 TeV. Since the gaugino partners are odd under matter-parity, an immediate interesting consequence of D-type SUSY-breaking models is that the s-gauginos are even under matter-parity and could be singly produced at the Large Hadron Collider (LHC).

Dirac gaugino masses are super-soft, and do not enter the renormalization group equations (RGEs) of the scalar soft masses. Ignoring all Yukawa couplings except for the top Yukawa coupling, the RGEs of all the soft s-fermion masses (except for the s-top masses) vanish at one-loop. The dominant two-loop contributions to the RGEs involve m_η^2 and are negative. Thus, if the soft masses are unified at the grand unified theory (GUT) scale, we would have a compact (compared to the typical models of SUSY-breaking such as gauge- and anomaly-mediated SUSY breaking) and inverted spectra with sleptons heavier than the squarks. In particular, the s-top would be the lightest sfermion and its mass can be approach current experimental bounds ($m_{\tilde{t}} > 300$ GeV) without s-leptons violating current experimental bounds ($m_{\tilde{l}} > 100$ GeV). Such spectra of D-type SUSY-breaking are very distinct from the typical MSSM spectra obtained by gauge-mediated supersymmetry breaking and other generic models of SUSY-breaking.

There are no trilinear soft terms in models of D-type SUSY-breaking, and the s-top masses can be as light as 400 GeV. While this may potentially solve the little hierarchy problem, where large radiative corrections to the soft Higgs mass $m_{H_u}^2$ requires a fine-tuning of a few percent to achieve successful EWSB, large $m_{\tilde{t}}$ and/or A_t are needed for the mass of the lightest CP-even boson to satisfy the CERN LEP bounds [19] of $m_h > 114.4$ GeV. Since we do not offer a complete model, we here give only a few remarks about EWSB with D-type SUSY-breaking.

One possibility of having successful EWSB is to extend the Higgs sector with an additional singlet chiral superfield, S , with the superpotential

$$\Delta W = \lambda S H_u H_d + \frac{\kappa}{3} S^3, \quad (7)$$

that replaces the $\Delta W = \mu H_u H_d$ term in the MSSM superpotential. While this superpoten-

tial of Eq. (7) is same as that of next-to-minimal supersymmetric Standard Model (NMSSM), unlike the typical NMSSM scenarios, we do not have trilinear SUSY-breaking terms in the potential. Instead, we include terms

$$\Delta\mathcal{L} = B_H(H_u H_d + \text{h.c.}) + B_S(S^2 + \text{h.c.}), \quad (8)$$

and still achieve successful EWSB with a lightest CP-even boson that satisfies the LEP2 bounds. It is worth emphasizing that, unless the fermionic component of S , the singletino (\tilde{s}), mixes significantly with the bino, our following analysis does not depend on the existence the chiral superfield S .

III. PSEUDO-DIRAC BINO DARK MATTER

The mass matrix of the neutral neutralino in a D-type SUSY-breaking scenario in the basis $(\lambda_1 \ \xi_1 \ \lambda_2 \ \xi_2 \ \tilde{H}_d \ \tilde{H}_u \ \tilde{s})$ is

$$\mathcal{M}_0 = \begin{pmatrix} 0 & M_1 & 0 & 0 & -\frac{g_Y}{2}v_d & \frac{g_Y}{2}v_u & 0 \\ M_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_2 & \frac{g_2}{2}v_d & -\frac{g_2}{2}v_u & 0 \\ 0 & 0 & M_2 & 0 & 0 & 0 & 0 \\ -\frac{g_Y}{2}v_d & 0 & \frac{g_2}{2}v_d & 0 & 0 & \mu & \frac{\lambda}{2}v_u \\ \frac{g_Y}{2}v_u & 0 & -\frac{g_2}{2}v_u & 0 & \mu & 0 & \frac{\lambda}{2}v_d \\ 0 & 0 & 0 & 0 & \frac{\lambda}{2}v_u & \frac{\lambda}{2}v_d & 2\frac{\kappa}{\lambda}\mu \end{pmatrix}, \quad (9)$$

where $M_{1,2}$ are the Dirac bino and wino mass, respectively. The gauge couplings of $U(1)_Y$ and $SU(2)_L$ SM gauge groups are denoted by g_Y and g_2 , respectively, $v_{u,d} = \sqrt{2}^{-1}\langle H_{u,d} \rangle$, and $\mu = \sqrt{2}^{-1}\lambda\langle S \rangle$. Since we are interested in the bino-dominated limit, we will assume that $m_2, \mu > m_1$. To simplify our analysis, we will also make these following three assumptions.

- First, the mass of the lightest bino state is smaller than the mass of the W -boson, M_W , so the only possible annihilation products are fermion-antifermion pairs. While the annihilation channels into the gauge and the Higgs bosons can be important for wino- and Higgsino-dominated χ_1^0 of the MSSM, the fermion-antifermion annihilation channels dominate the total annihilation cross section in the bino-dominated χ_1^0 even

when the gauge boson annihilation channels are kinematically allowed [2]. For the D-type SUSY-breaking scenario, we will simply assume this and postpone the verification of this assumption in a later study.

- Second, we assume that M_1 , M_2 and μ are all positive. While the relative signs and phases of these parameters are important when making a detailed study, we will assume this simple case.
- Third, the matrix \mathcal{M}_0 has the hierarchy

$$\mu \gg m_1 \sim m_2 \sim v_{u,d}, \quad (10)$$

so we can expand in μ^{-1} and keep the lowest terms. However, we do not assume that m_1 and m_2 are nearly-degenerate, so there are no co-annihilation contributions from the charged winos.

With these three assumptions, we first compute the relic density in the limit of pure Dirac bino ($\mu \rightarrow \infty$), and then compute the corrections induced by EWSB to first-order in the effects of EWSB and μ^{-1} . We then compute the direct detection cross section of pure- and pseudo-Dirac bino to the same order.

A. Relic density in the pure Dirac bino limit

In the limit of large μ , the Higgsinos and the singletino decouple and the lightest neutralino state is a pure Dirac bino. In terms of two-component Weyl spinors, we have the following Lagrangian of the Dirac bino mass and bino-fermion-sfermion interactions

$$\Delta\mathcal{L} = -\sqrt{2}g_Y Y_L(\lambda_1 q_L \tilde{q}_L^* + \lambda_1^\dagger q_L^\dagger \tilde{q}_L) - \sqrt{2}g_Y Y_R(\lambda_1 \overline{q}_R \tilde{q}_R^* + \lambda_1^\dagger \overline{q}_R^\dagger \tilde{q}_R) - M_1(\lambda_1 \chi_1 + \lambda_1^\dagger \chi_1^\dagger), \quad (11)$$

where q_L and \overline{q}_R are two-component SM fermion with hypercharge Y_L and Y_R , respectively. We define the Dirac spinors

$$Q = \begin{pmatrix} q_L \\ \overline{q}_R^\dagger \end{pmatrix}, \quad D = \begin{pmatrix} \lambda_1 \\ \chi_1^\dagger \end{pmatrix}, \quad D' = C\overline{D}^T = \begin{pmatrix} \chi_1 \\ \lambda_1^\dagger \end{pmatrix}, \quad (12)$$

and the projection operators

$$P_L = \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (13)$$

We can then rewrite the Lagrangian in terms of the Dirac spinors

$$\mathcal{L} = -\sqrt{2}g_Y Y_L (\overline{D}' P_L Q \tilde{q}_L^* + \overline{Q} P_R D' \tilde{q}_L) - \sqrt{2}g_Y Y_R (\overline{D} P_R Q \tilde{q}_R^* + \overline{Q} P_L D \tilde{q}_R) - M_1 \overline{D} D. \quad (14)$$

Integrating out the sfermions we obtain the effective four-fermion interactions

$$\mathcal{L}_{\text{eff}} = \frac{2g_Y^2 Y_L^2}{M_{\tilde{q}_L}^2} (\overline{D}' P_L Q) (\overline{Q} P_R D') + \frac{2g_Y^2 Y_R^2}{M_{\tilde{q}_R}^2} (\overline{D} P_R Q) (\overline{Q} P_L D). \quad (15)$$

Applying Fierz transformation, we obtain

$$\mathcal{L}_{\text{eff}} = \frac{g_Y^2 Y_L^2}{M_{\tilde{q}_L}^2} (\overline{D}' \gamma^\mu P_L D') (\overline{Q} \gamma_\mu P_R Q) + \frac{g_Y^2 Y_R^2}{M_{\tilde{q}_R}^2} (\overline{D} \gamma^\mu P_R D) (\overline{Q} \gamma_\mu P_L Q), \quad (16)$$

which will be useful when we compute the direct-detection rate in the limit of pure Dirac bino dark matter.

From the effective interactions of Eqs. (15) and (16), we have the thermal-averaged annihilation cross section

$$\langle \sigma(\overline{D} D \rightarrow \overline{f} f) v \rangle = \frac{g_Y^4 M_1^2}{8\pi} \sum_f \frac{N_f Y_f^4}{M_f^4} \left(1 + \mathcal{O}\left(\frac{T}{M_1}\right) \right), \quad (17)$$

where v is the relative velocity of the annihilating binos and the summation sums over all the fermions of the SM except for the top quark, N_i is the color factor ($N = 3$ for quarks and $N = 1$ for leptons), and T is the temperature of Dirac bino. Since this annihilation cross section is not p -wave suppressed, it is a good approximation to keep the leading, temperature-independent, contribution, as we have done here. The relic density of the pure Dirac bino is then given by [20]

$$\Omega h^2 = 2 \frac{x_F}{\sqrt{g_*}} \frac{8.7 \times 10^{-11} \text{ GeV}^{-2}}{\langle \sigma(\overline{D} D \rightarrow \overline{f} f) v \rangle}, \quad (18)$$

where $g_* = 96$ is the number of relativistic degrees of freedom at the freeze-out temperature T_F , and $x_F = M_1/T_F$. Also, in Eq. (18), we have included a factor of 2 to account for the relic density of both the particle *and* the antiparticle, as explained in the Appendix of Ref. [21]. In general, the freeze-out temperature of species A with mass M_A is given by iteratively solving the formula

$$x_F = \ln \left(\frac{5}{4} \sqrt{\frac{45}{8}} \frac{d_A}{2\pi^3} \frac{M_A M_{\text{Pl}}}{\sqrt{g_* x_F}} \langle \sigma(AA \rightarrow XX) v_F \rangle \right), \quad (19)$$

where d_A is the degrees of freedom of A , and $\langle\sigma(AA \rightarrow XX)v_F\rangle$ is the thermal-averaged cross section evaluated at the freeze-out temperature

$$\langle\sigma(AA \rightarrow XX)v_F\rangle \equiv \langle\sigma(AA \rightarrow XX)v\rangle|_{T \rightarrow T_F}. \quad (20)$$

As a comparison, the relic density of a pure Majorana bino in the MSSM is (see Reference [10], for example)

$$\Omega h^2 \simeq 2 \frac{x_F}{\sqrt{g_*}} \frac{8.7 \times 10^{-11} \text{ GeV}^{-2}}{\langle\sigma(\tilde{B}\tilde{B} \rightarrow \tilde{f}\tilde{f})v_F\rangle}, \quad (21)$$

where

$$\langle\sigma_{\tilde{B}\tilde{B}}v_F\rangle = \frac{g_Y^4}{2\pi} \sum_f N_f Y_f^4 \frac{r_f(1+r_f^2)}{M_{\tilde{f}}^2(1+r_f)^4 x_F}, \quad \text{with} \quad r_f \equiv \frac{M_1^2}{M_{\tilde{f}}^2}, \quad (22)$$

is the thermal-averaged annihilation cross section evaluated at the freeze-out temperature T_F , which can be solved from Eq. (19).

In Figure 1, we plot Eqs. (18) and (21) as functions of a common scalar soft mass M_{SUSY} , as well as the relic density calculated by MicrOMEGAs 2.0 [22] as checks for sample spectra that approach the bino-dominated limit. Although neither results are consistent with the WMAP observational bounds of Eq. (2), we see that the relic density of a pure Dirac bino is smaller by roughly a factor of 4 compared to that of the Majorana bino, and there may be less fine tuning in the D-type SUSY breaking models than the MSSM to obtain the observed relic density of dark matter.

B. Relic density of pseudo-Dirac bino

Because of EWSB contributions, the D-type SUSY-breaking spectra has a pseudo-Dirac bino consisting of two nearly-degenerate bino states when $\mu \gg M_1, M_2$. Expanding the effective bino mass matrix to order $\mathcal{O}(\mu^{-1})$, we have

$$\mathcal{M}_{\text{bino}} = \begin{pmatrix} g_Y^2 \frac{v_u v_d}{2\mu} & M_1 \\ M_1 & 0 \end{pmatrix}, \quad (23)$$

giving the mass eigenstates

$$\chi_{1,2}^0 = \frac{1}{\sqrt{2}}(\lambda_1 \mp \xi_1), \quad (24)$$

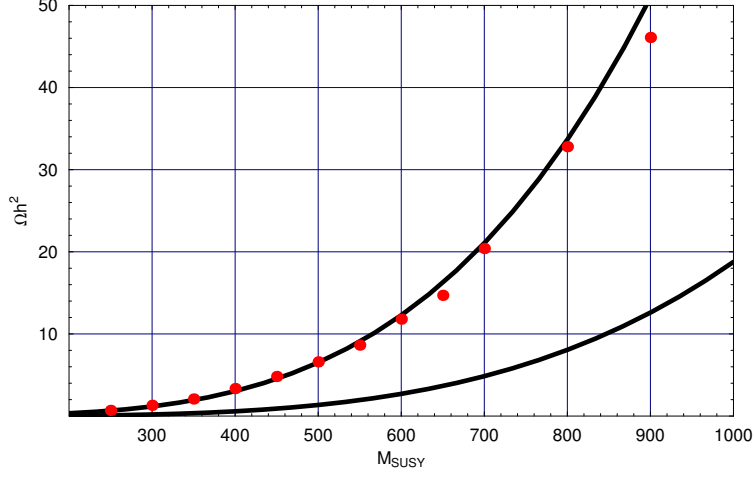


FIG. 1: The relic densities Ωh^2 of pure Dirac (lower line) and Majorana (upper line) bino as a function of a common sfermion mass M_{SUSY} . The dots on top of the upper line are computed using MicrOMEGAs 2.0 with spectra whose χ_1^0 is mostly the Majorana bino.

with masses

$$|M_{\chi_{1,2}^0}| = M_1 \mp \frac{g_Y^2 v_u v_d}{4\mu}, \quad (25)$$

where we have used the assumption that $M_1 > 0$. The gauge interactions of Eq. (14) can now be written as

$$\begin{aligned} \Delta\mathcal{L} = & -g_Y Y_L (\chi_1 q_L \tilde{q}_L^* + \chi_1^\dagger q_L^\dagger \tilde{q}_L) - g_Y Y_R (\chi_1 \bar{q}_R \tilde{q}_R^* + \chi_1^\dagger \bar{q}_R^\dagger \tilde{q}_R) \\ & - g_Y Y_L (i\chi_2 q_L \tilde{q}_L^* - i\chi_2^\dagger q_L^\dagger \tilde{q}_L) - g_Y Y_R (i\chi_2 \bar{q}_R \tilde{q}_R^* - i\chi_2^\dagger \bar{q}_R^\dagger \tilde{q}_R), \end{aligned} \quad (26)$$

where we have made a rotation $\chi_2 \rightarrow i\chi_2$ so that its mass appears in the Lagrangian with a positive sign. Up to a factor of $\sqrt{2}^{-1}$ in the couplings, both χ_1^0 and χ_2^0 have the interactions similar to the bino of the MSSM.

Integrating out the s-fermions, we have the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{g_Y^2 Y_L^2}{M_{\tilde{Q}_L}^2} [(\bar{\mathcal{N}}_1 P_L Q)(\bar{Q} P_R \mathcal{N}_1) + (\bar{\mathcal{N}}_2 P_L Q)(\bar{Q} P_R \mathcal{N}_2) \\ & - i(\bar{\mathcal{N}}_1 P_L Q)(\bar{Q} P_R \mathcal{N}_2) + i(\bar{\mathcal{N}}_2 P_L Q)(\bar{Q} P_R \mathcal{N}_1)] \\ & + \frac{g_Y^2 Y_R^2}{M_{\tilde{Q}_R}^2} [(\bar{\mathcal{N}}_1 P_R Q)(\bar{Q} P_L \mathcal{N}_1) + (\bar{\mathcal{N}}_2 P_R Q)(\bar{Q} P_L \mathcal{N}_2) \\ & - i(\bar{\mathcal{N}}_1 P_R Q)(\bar{Q} P_L \mathcal{N}_2) + i(\bar{\mathcal{N}}_2 P_R Q)(\bar{Q} P_L \mathcal{N}_1)], \end{aligned} \quad (27)$$

where \mathcal{N}_i (for $i = 1, 2$) are the four-component Majorana spinor

$$\mathcal{N}_i \equiv \begin{pmatrix} \chi_i \\ \chi_i^\dagger \end{pmatrix}. \quad (28)$$

Although χ_1^0 is the lightest state, it is nearly degenerate with χ_2^0 with a fractional mass difference of

$$\Delta \equiv \frac{M_{\chi_2^0} - M_{\chi_1^0}}{M_{\chi_2^0}} = \frac{g_Y^2 v_u v_d}{2M_1 \mu}, \quad (29)$$

so the difference in masses between χ_2^0 and χ_1^0 is naturally only a few GeV's. (For example, $\Delta = 0.04$ for $M_1 = 75$ GeV, $\mu = 500$ GeV, and $\tan \beta = 5$.) The relic density now depends on processes involving χ_2^0 , as χ_2^0 may be abundant when χ_1^0 freezes out [23]. In particular, we have to consider the self- and co-annihilation processes involving χ_2^0 in addition to the self annihilation of χ_1^0 .

Since $\chi_{1,2}^0$ have interactions similar to the Majorana bino of the MSSM up to a factor in the couplings, their self annihilation cross sections are of the same form as Eq. (22)

$$\sigma(\chi_1^0 \chi_1^0 \rightarrow \bar{f} f) = \frac{g_Y^4}{8\pi} \sum_f N_f Y_f^4 \frac{r_{1f}(1 + r_{1f}^2)}{M_{\tilde{f}}^2(1 + r_{1f})^4} v, \quad r_{1f} \equiv \frac{M_{\chi_1^0}^2}{M_{\tilde{f}}^2}, \quad (30)$$

with a similar formula for the annihilation of χ_2^0 . As with the case of the MSSM bino, the self annihilation cross sections of both χ_1^0 and χ_2^0 are p -wave suppressed when thermal-averaged. The co-annihilation cross section is given by

$$\sigma(\chi_1^0 \chi_2^0 \rightarrow \bar{f} f) v = \frac{g_Y^4}{32\pi} (M_{\chi_1^0} + M_{\chi_2^0})^2 \sum_f N_f \frac{Y_f}{M_{\tilde{f}}^4}. \quad (31)$$

Note that this cross section reduces to the annihilation cross section of the pure Dirac bino in Eq. (17) when $M_{\chi_2^0} = M_{\chi_1^0}$.

We are now ready to calculate the dark matter relic density taken into account effects of co-annihilation. We first define

$$\sigma_{\text{eff}} = \frac{4}{g_{\text{eff}}^2} \left(\sigma_{\chi_1^0 \chi_1^0} + 2\sigma_{\chi_2^0 \chi_1^0} (1 + \Delta)^{3/2} e^{-x\Delta} + \sigma_{\chi_2^0 \chi_2^0} (1 + \Delta)^3 e^{-2x\Delta} \right), \quad (32)$$

where $\Delta = (m_{\chi_2^0} - m_{\chi_1^0})/m_{\chi_1^0}$, and

$$g_{\text{eff}} = 2 + 2(1 + \Delta)^{3/2} e^{-x\Delta}. \quad (33)$$

The relic density of the lightest Majorana state is now

$$\Omega h^2 = \frac{x_F}{\sqrt{g_*}} \frac{8.7 \times 10^{-11} \text{ GeV}^{-2}}{I_a + 3I_b/x_F} \quad (34)$$

where

$$I_a = x_F \int_{x_F}^{\infty} \frac{dx}{x^2} a_{\text{eff}}(x), \quad \text{and} \quad I_b = 2x_F \int_{x_F}^{\infty} \frac{dx}{x^3} b_{\text{eff}}(x). \quad (35)$$

The functions a_{eff} and b_{eff} are the coefficients of $\sigma_{\text{eff}} v$ expanded in v^2 ,

$$\sigma_{\text{eff}} v = a_{\text{eff}} + b_{\text{eff}} v^2, \quad (36)$$

and the freeze-out temperature is solved by the formula similar to Eq. (20)

$$x_F = \ln \left(\frac{5}{4} \sqrt{\frac{45}{8}} \frac{g_{\text{eff}}}{2\pi^3} \frac{M_{\chi_1^0} M_{\text{Pl}}}{\sqrt{g_*} x_F} (a_{\text{eff}} + 6x_F^{-1} b_{\text{eff}}) \right), \quad (37)$$

with g_{eff} , a_{eff} , and b_{eff} now evaluated at the freeze-out temperature.

It is important to note that the relic density of pseudo-Dirac bino reduces in the pure Dirac bino limit correctly. Since the self-annihilation cross sections of χ_1^0 and χ_2^0 are p -wave suppressed, we can make the approximation that

$$\sigma_{\text{eff}} \simeq \frac{8}{g_{\text{eff}}^2} \sigma_{\chi_2^0 \chi_1^0} (1 + \Delta)^{3/2} e^{-x_F \Delta}, \quad (38)$$

which is valid as long as the exponential Boltzmann suppression $e^{-x_F \Delta}$ is much larger than p -wave suppression of the self annihilation x_F^{-1} . (For example, for $\mu = 500 \text{ GeV}$, $\tan \beta = 5$, and $x_F = 20$, we have $e^{-x_F \Delta} \sim 0.66$ while $x_F^{-1} = 0.05$, so the approximation is valid.) In the Dirac bino limit of $\Delta \rightarrow 0$, the effective cross section $\sigma_{\text{eff}} v$ is half of the annihilation of pure Dirac bino in Eq. (17) because of the factor $g_1 g_2 g_{\text{eff}}^{-2}$ in $\sigma_{\text{eff}} v$ approaches $\frac{1}{2}$, naively leading to a relic density that is twice as large as the pure Dirac bino. However, in the case of the pure Dirac bino, there is an additional factor of 2 in its relic density to account for both the particle and antiparticle, and the relic density of pseudo-Dirac bino approaches that of the pure-Dirac bino correctly.

We can also find the leading dependence of the relic density of the pseudo-Dirac bino on the splitting in mass Δ . For small Δ (such that $e^{x_F \Delta} \gg x_F^{-1}$), where the main annihilation mode of the $\chi_2^0 - \chi_1^0$ system is the co-annihilation mode, the annihilating cross section is suppressed compared to the annihilation in the limit of pure Dirac bino by a factor of

$$\frac{I_a}{\langle \sigma(\overline{D} D \rightarrow \overline{f} f) v \rangle} = 2 \frac{e^{-x_F \Delta}}{[1 + (1 + \Delta)^{3/2} e^{-x_F \Delta}]^2} \left(1 + \frac{5}{2} \Delta + \mathcal{O}(\Delta^2) \right), \quad (39)$$

when we expand I_a in Δ . The relic density of the pseudo-Dirac bino increases correspondingly by (taking into account the factor of 2 in the relic density of pure Dirac bino)

$$\frac{\Omega_{\mathcal{N}}}{\Omega_D} = \frac{1}{4} (e^{x_F \Delta}) [1 + (1 + \Delta)^{3/2} e^{-x \Delta}]^2 (1 - \frac{5}{2} \Delta + \mathcal{O}(\Delta^2)), \quad (40)$$

and we can explicitly see the relic density of pseudo-Dirac bino reduces correctly in the pure Dirac bino limit ($\Delta \rightarrow 0$).

In Figure 2, we plot the relic density of pseudo-Dirac bino as a function of a common scalar soft mass M_{SUSY} for several values of Δ . We see that, even for $\Delta = 0.05$, the relic density of the pseudo-Dirac bino is still less than the Majorana bino by about a factor of 2. For $\Delta = 0.10$, the relic density of the pseudo-Dirac bino is about the same, though slightly larger, as that of the MSSM bino. For $\Delta = 0.15$, the pseudo-Dirac bino relic density is larger than that of the MSSM bino by about a factor of 3, signalling the decreasing effects of co-annihilation and the weaker interactions between matter and the lighter pseudo-Dirac bino state χ_1^0 compared to the MSSM bino.

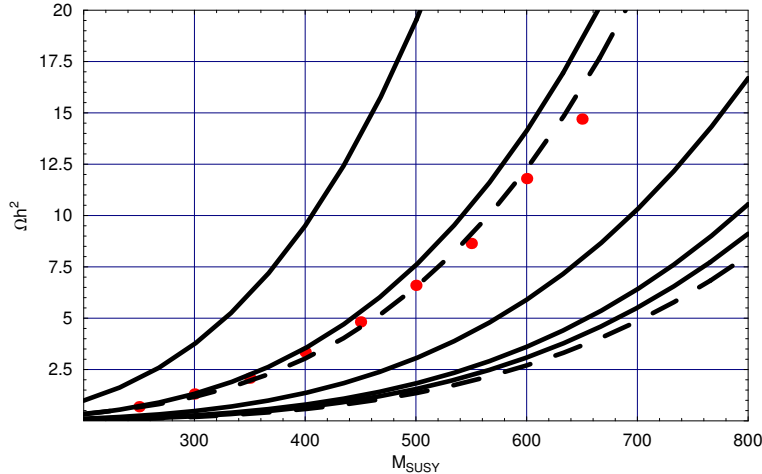


FIG. 2: The dashed lines are those in Figure 1. The solid lines, from bottom to top, correspond to the relic densities of the pseudo-Dirac plot for $\Delta = 0.01, 0.02, 0.05, 0.10$, and 0.15 .

C. Direct detection in the pure Dirac bino limit

The direct detection experiments [13][14] measure recoils of heavy nuclei from interactions with dark matter. The recoil energies are of the scale of tens of keV, and the bounds are

expressed in terms of elastic cross sections between dark matter and the nucleon. The most stringent bounds set by these experiments come from the spin-independent interactions between dark matter and the nuclei, and it is only those interactions that we consider for the pure Dirac bino.

To compute the elastic cross section between dark matter and the nucleon, we re-write the effective interaction of Eq. (16) as vector and axial-vector interactions

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \left[a_L (\overline{D}' \gamma^\mu P_L D') + a_R (D \gamma^\mu P_R D) \right] \overline{Q} \gamma_\mu Q \\ & + \left[a_L (\overline{D}' \gamma^\mu P_L D') - a_R (D \gamma^\mu P_R D) \right] \overline{Q} \gamma_\mu \gamma^5 Q,\end{aligned}\quad (41)$$

where

$$a_{L,R} = \frac{g_Y^2 Y_{L,R}^2}{2M_{\tilde{Q}_{L,R}}^2}.\quad (42)$$

As vector contributions of the quarks in the nucleus add coherently, we can express the cross section between Dirac bino and a nucleus $N(Z, A)$ as [3]

$$\sigma_{\text{vec}}^N = \frac{b_N^2}{\pi} \frac{M_1^2 m_N^2}{(M_1 + m_N)^2},\quad (43)$$

with $b_N = Zb_p + (A - Z)b_n$, $b_p = 2b_u + b_d$, $b_n = b_u + 2b_d$, and

$$b_u = \frac{1}{2}(a_{u_L} + a_{u_R}) = \frac{g_Y^2}{4} \left(\frac{1}{36M_{\tilde{u}_L}^2} + \frac{4}{9M_{\tilde{u}_R}^2} \right),\quad (44)$$

$$b_d = \frac{1}{2}(a_{d_L} + a_{d_R}) = \frac{g_Y^2}{4} \left(\frac{1}{36M_{\tilde{d}_L}^2} + \frac{1}{9M_{\tilde{d}_R}^2} \right).\quad (45)$$

The experimental bounds are expressed in the bino-nucleon cross section σ_n that is related to the bino-nucleus cross section σ_N by

$$\sigma_{\text{vec}}^n = \frac{M_n}{M_N} \frac{(M_1 + M_N)^2}{(M_1 + M_n)^2} \frac{\sigma_{\text{vec}}^N}{A^2}.\quad (46)$$

In the simplified case where all the sfermion masses are degenerate with a common mass M_{SUSY} , for the ^{73}Ge detector used in CDMS II [13], the bino-nucleon cross section is

$$\sigma_{\text{vec}}^n = (8.6 \times 10^{-39}) \left(\frac{500 \text{ GeV}}{M_{\text{SUSY}}} \right)^4 \text{ cm}^2.\quad (47)$$

This is well above the upper-bound of $2 \times 10^{-43} \text{ cm}^2$ set by CDMS II for dark matter with mass on the order of 100 GeV, and the limit of pure Dirac bino with mass of the scale of 100 GeV is ruled out as dark matter.

D. Direct detection of pseudo-Dirac bino

As stated in the Introduction, as long as the splitting between the two states of the pseudo-Dirac bino is larger than 10s of keVs, the direct detection experiments are only sensitive to the lighter state χ_1^0 . Since χ_1^0 in our approximation behaves exactly as the MSSM bino up to a scaled coupling, the direct detection bounds are similar to the case of the MSSM bino. The direct detection rates of the MSSM neutralino has been studied extensively in the literature [1][3][4][5][7]. In particular, being a Majorana particle, there is no longer a vector interaction with the quarks, and the resulting χ_1^0 -nucleon cross section is much smaller. Here we will simply state the results from the literature for the direct detection rates for the MSSM bino \tilde{B} (for the pseudo-Dirac bino χ_1^0 , simply make the replacement $g_Y \rightarrow \sqrt{2}^{-1}g_Y$). Our presentation here is mainly based on Reference [5].

The four-fermion effective Lagrangian for the bino \tilde{B} is given by

$$\mathcal{L} = \frac{g_Y^2}{2}(\tilde{B}\gamma^\mu\gamma_5\tilde{B})\left[\frac{Y_L^2}{M_{\tilde{Q}_L}^2}(\overline{Q}\gamma_\mu P_R Q) - \frac{Y_R^2}{M_{\tilde{Q}_R}^2}(\overline{Q}\gamma_\mu P_L Q)\right], \quad (48)$$

since $\overline{\tilde{B}}\gamma_\mu\tilde{B} = 0$, there are only axial-vector interactions with the coefficients

$$A_Q = \frac{g_Y^2}{4}\left(\frac{Y_L^2}{M_{\tilde{Q}_L}^2} + \frac{Y_R^2}{M_{\tilde{Q}_R}^2}\right). \quad (49)$$

The evaluation of the elastic cross section will now require the matrix elements

$$\langle n|\overline{Q}\gamma_\mu\gamma_5 Q|n\rangle = 2s_\mu^n\Delta_Q^n, \quad (50)$$

where s_μ^n is the spin of the nucleon n , and Δ_Q^n (extracted from experiments) is the fraction of nucleon spin carried by quark Q . The experimental values are [24]

$$\begin{aligned} \Delta_u^p &= 0.77, & \Delta_d^p &= -0.38, & \Delta_s^p &= -0.09, \\ \Delta_u^n &= -0.38, & \Delta_d^n &= 0.77, & \Delta_s^n &= -0.09. \end{aligned} \quad (51)$$

The elastic cross section is then

$$\sigma_{\text{axial-vec}}^N = \frac{16}{\pi} \frac{M_{\chi_1^0}^2 M_N^2}{(M_{\chi_1^0}^2 + M_N^2)^2} \frac{J+1}{J} \left(\langle S_p \rangle \sum_{u,d,s} (A_Q \Delta_Q^p) + \langle S_n \rangle \sum_{u,d,s} (A_Q \Delta_Q^n) \right)^2, \quad (52)$$

where J is the spin of the nucleus, $\langle S_{p,n} \rangle = \langle N|S_{p,n}|N \rangle$ are the expectation values of the spin content of the proton and neutron groups in the nucleus, respectively. Their values

values $\langle S_{p,n} \rangle$ for ^{73}Ge are given by the shell model as [25]

$$\langle S_p \rangle_{\text{Ge}} = +0.011, \quad \langle S_n \rangle_{\text{Ge}} = -0.491. \quad (53)$$

For ^{73}Ge ($J = \frac{9}{2}$), $M_{\chi_1^0} = 75$ GeV, and a common squark mass of M_{SUSY} , the spin-dependent cross section is then

$$\sigma_{\text{axial-vec}}^N = 1.0 \times 10^{-42} \left(\frac{500 \text{ GeV}}{M_{\text{SUSY}}} \right)^4 \text{ cm}^2, \quad (54)$$

which is consistent with the CDMS II upper bounds of $1 \times 10^{-38} \text{ cm}^2$. It should be noted, however, that Higgsino components of χ_1^0 that we ignore here may change the direct detection rates significantly. The Higgsinos have scalar interactions with nucleus, which are coherent and proportional to the nucleus mass. If Higgsino composition of χ_1^0 are significant, the spin-independent cross section may overwhelm the spin-dependent cross section. We will leave this for future work.

IV. CONCLUSIONS

In this paper we have calculated the relic density and direct detection rates for pseudo-Dirac bino, which arises naturally as dark matter in supersymmetric models with D -type SUSY-breaking. Although we have performed these calculations in some very simple limits, our results are nonetheless interesting. For small mass splitting between the two pseudo-Dirac bino states (of a few percent in the fractional difference in masses), the relic density of pseudo-Dirac bino is closer to WMAP observations compared to the MSSM bino, while its direct detection rate is smaller than the MSSM bino by a factor of 4. The reduced relic density of the pseudo-Dirac bino implies that there may be less fine-tuning of the D -type SUSY-breaking spectra to achieve a dark matter relic density consistent with observations.

As with the rich phenomenology of the neutralino sector of the MSSM, relaxing any of the assumptions of this study can lead to significantly different conclusions. In particular, it would be interesting to include annihilation to the gauge and Higgs bosons. Also, the relative signs between the various mass parameters can be important, as well as the wino/Higgsino mixture of χ_1^0 . In addition, although qualitatively there may be less fine-tuning to achieve observed relic density, it is important to quantify the degree of fine-tuning and compare it with the MSSM. We leave these open projects for future work.

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